The first thing to observe is that your total winnings (in hundreds) are given by

$$h - (2^r - 1)$$

where h is the total number of heads that come up, and r is the length of the run of tails at the end of this game. (So r could be 0, if the last coinflip is a head.) For example, suppose you played a 10-flip game and got

$$HHTHTHHTTT$$

Then $h = 5$, and $r = 3$, because you end with a string of 3 tails. In this game, you'd end up with total winnings of

$$5 - (2^3 - 1) = 5 - 7 = -2.$$

In other words, you end the game down \$200; the \$700 you lose on the last three flips wipes out the \$500 you spent the first seven flips amassing.

Suppose you flip the coin a thousand times. The expected value of h (that is, the expected number of heads) is 500. The probability that h is within a few percentage points of 500 is very high, so to simplify matters, let's suppose that h is 500 on the nose.

Now what about r? The chance that $r = 0$ is just the chance that the last flip is a head, or 1/2. The chance that $r = 1$ is the chance that the last two flips are a tail followed by a head; that happens 1/4 of the time. The chance that $r = 2$ is 1/8, that $r = 3$ is 1/16, and so on.

If r is less than 9, then $2^r - 1$ is at most 255, and so you finish the game ahead. And the probability that $r = 0, 1, 2, 3, 4, 5, 6, 7$, or 8 is

$$1/2 + 1/4 + 1/8 + \ldots + 1/512 = 511/512.$$

In other words, there's less than a 1 in 500 chance of finishing the game in the red! Sounds good – until you think about just how much you stand to lose. The chance that $r = 10$ is 1 in 2048, not at all out of the realm of possibility. And if $r = 10$, your total take is

$$500 - (2^{10} - 1) = 500 - 1023 = -523$$

In other words, you're out more than \$50,000. And with each extra tail, your loss grows exponentially. There's better than a one in a hundred thousand chance – leagues more likely than winning the lottery – that r is at least 16; and if that's the case, you're going to lose more than \$6 million dollars. Note that the *most* you can win playing the martingale – even if you get heads every single time! – is \$100,000. Losses are highly unlikely – but if you do lose, your losses are massive compared to the potential gain.

And it gets worse – because if the stake you're playing with is less than \$6 million, the relevant thing isn't just whether you *end up* deeper than that in the hole; if you hit your bad string of 16 heads anywhere along the way, you're busted. In fact, if you play long enough, you're all but certain to go broke eventually; even though within any fixed period you're likely to come out ahead! This phenomenon is called, for obvious reasons, *gambler's ruin.*